Book Review

Georg Glaeser, Hellmuth Stachel, Boris Odehnal: *The Universe of Conics.*
Springer Spektrum, Heidelberg 2016, 488 p., 350 illus. in color, ISBN 978-3-662-45449-7, 978-3-662-45450-3 (eBook).

This book presents conics in a remarkable variety of contexts, including the Euclidean plane, Euclidean space, and projective, affine, spherical, and differential geometry, as well as physical and technical applications. Substantial results, from classical to current, are derived in each context, and additional results are described. The authors unify the material by presenting some theorems in more than one context. They prove theorems efficiently by using a wide range of synthetic and analytic approaches.

The text includes hundreds of beautifully designed figures. Making key geometric relationships clear, many figures are essential to proofs and lead to notable geometric constructions. Other figures illustrate theorems dramatically.

The first third of the book uses elementary Euclidean and differential geometry to study conics in $\mathbb{R}^2$ and $\mathbb{R}^3$. It proceeds without delay from standard results to remarkable ones. For example, one logical chain starts with a nice kinematic derivation of the reflection properties of conics. A corollary states that a circle is formed by the feet of the perpendiculars from the foci of an ellipse or hyperbola to the tangent lines. It follows that an ellipse or hyperbola is formed by the centers of all circles that contain a given point and are are tangent to a given circle. Analyzing Dandelin's construction of conics as sections of cones shows the following: the apexes of all right circular cones having a given conic as a section form a conic focal with the first; that is, the two conics lie in orthogonal planes, and the vertices of each conic are the foci of the other. All these results lead to the study of a Dupin cyclide, which is the envelope of all spheres with equators tangent to two circles in a plane.

The middle half of the book develops projective properties of conics. Proofs of basic properties are included, but prior acquaintance with projective geometry would probably help a reader, and results on algebraic curves are used on occasion. The authors stress connections between conics and polarities and emphasize the role of matrix transformations. Among the high points are generalized forms of Desargues's Involution Theorem and MacLaurin's theorem that infinitely many triangle can be inscribed in one conic and circumscribed about another when any such triangle exists.

A remaining chapter concerns the use of affine transformations in studying conics. The final chapter considers conics in spherical and other non-Euclidean geometries.

Overall, the book contains a wide range of striking results about conics. Numerous figures highlight geometry as a mathematical and visual art.

Robert Bix
Mathematics Department
University of Michigan-Flint
bobbix@umflint.edu